**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Answer:** - Mean(µ)= 45 minutes

Standard Deviation(σ) = 8 Minutes

Because the car is dropped off 10 minutes before work begins, the remaining time for servicing is 60 minutes - 10 minutes = 50 minutes.

Z-score Z = (X - μ) / σ X = remaining time available = 50 minutes Z = (50 - 45) / 8 = 0.625

probability associated with the Z-score of 0.625 using a conventional normal distribution table or statistical software. Assume that P (Z > 0.625) = 0.2676.

As a result, **the correct answer is B. 0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Answer: -**

Value - Mean/(SD) = Z score

Z score for 44 is (44 - 38)/6 = 1 corresponding to 84.13%.

People above the age of 44 = 100 - 84.13 = 15.87% 63 of 400

Z score for 38 is equal to (38 − 38)/6, or 0 => 50%.

Hence Age group 38–44 = 84.13–50 = 34.13%, or 137 out of 400

**So, False A. More employees at the processing center are older than 44 than between 38 and 44.**

36 out of 400 is the Z score for 30 (30 - 38)/6 = -1.33 = 9.15 %.

**Hence True B. training program for employees under the age of 30 at the center would be expected to attract about 36 employees - TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Answer: -**

Distributions:

2X1: 2X1's distribution is an enlarged version of X1's distribution. The distribution will still be normal, but with a mean of 2 (twice X1's mean) and a variance of 4 (four times X1's variance).

X1 + X2: X1 and X2 added together will also have a normal distribution. The total will have a mean of 2 (the sum of the means of X1 and X2) and a variance of 2 (the sum of the variances of X1 and X2) because X1 and X2 are independently and identically distributed (iid).

Parameters:

X1: The mean of X1 is 1, while the mean of 2X1 is 2, which is twice that. Four times the volatility of X1 is the variance of 2X1, which has a variance of 42.

X1 + X2: The average of X1 + X2 is equal to 2, which is the sum of X1 and X2's means. The sum of the variances of X1 and X2 is 22, which is the variance of X1 + X2.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Answer: -** Given:

Mean (μ) = 100

Standard deviation (σ) = √400 ≈ 20

A cumulative probability of 0.99 corresponds to a z-score of roughly 2.33.

Now, we can use the formula to determine the values of a and b:

a = μ - z \* σ

b = μ + z \* σ

Making the following substitutions:

a = 100 - 2.33 \* 20 ≈ 100 - 46.6 ≈ 53.4

b = 100 + 2.33 \* 20 ≈ 100 + 46.6 ≈ 146.6

As a result, 53.4 and 146.6, respectively, are the values of a and b that are symmetric about the mean and at which the chance of the random variable taking a value between them is 0.99.

The right response is **D. 48.5, 151.5**.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Answer: -**

A. The profit distributions must be converted from dollars to rupees using the current exchange rate of $1 = 45.

Given:

Profit1 ~ N (5, 9) (in $ Million)

Profit2 ~ N (7, 16) (in $ Million)

Exchange rate: $1 = Rs. 45

multiply the mean and standard deviation of each distribution by the conversion rate in order to convert the profit distributions to Rupees.

Profit1 in Rupees: N (5 \* 45, 9 \* 45^2) = N (225, 182250) (in Rs. Million)

Profit2 in Rupees: N (7 \* 45, 16 \* 45^2) = N (315, 129600) (in Rs. Million)

1. must locate the associated z-scores for a 95% confidence interval in order to designate a rupee range based on the mean that contains 95% likelihood for the company's annual profit. A 95% confidence interval has a z-score of roughly 1.96.

For Profit1:

Rupee range: (225 - 1.96 \* √182250, 225 + 1.96 \* √182250) ≈ (141.05, 308.95) (in Rs. Million)

For Profit2:

Rupee range: (315 - 1.96 \* √129600, 315 + 1.96 \* √129600) ≈ (226.68, 403.32) (in Rs. Million)

1. compare the means of the profit distributions in Rupees to identify which division has a higher likelihood of posting a loss in any given year.

Profit1 mean in Rupees: 225 (in Rs. Million)

Profit2 mean in Rupees: 315 (in Rs. Million)

Profit1 is more likely to experience a loss in a given year because it has a smaller mean than Profit2, which has a larger mean.

The following are the questions' answers:

A. The range in rupees is (141.05, 308.95).

B. 187.79 is the 5th percentile of profit in rupees.

C. Profit1 division is more likely to experience a loss in any given year.